Supplementary Paper for:

Nonparametric Testing for Differences in Electricity Prices: The Case of the Fukushima Nuclear Accident

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Our basic assumptions, are essentially equivalent to those in Ruppert and Wand (1994) with some straightforward adjustments to our functional data and time series context.

A1 (Asymptotic Scenario) $nm \to \infty$, where $m = m_n \geq 2$ such that $m_n \asymp n^{\theta}$ with $0 \leq \theta < \infty$. Hereby, "$m_n \asymp n^{\theta}$" denotes that the two sequences m_n and n^{θ} are asymptotically equivalent, i.e., that $\lim_{n\to\infty}(m_n/n^{\theta}) = C$ with constant $0 < C < \infty$.

A2 (Random Design) The triple (Y_{ij}, U_{ij}, Z_i) has the same distribution as (Y, U, Z) with pdf f_{YUZ} where $f_{YUZ}(y, u, z) > 0$ for all $(y, u, z) \in \mathbb{R} \times [0, 1]^2$ and zero else. The error term ϵ_{ij} is iid and independent from X^c_s, $U_{s\ell}$, and Z_s for all $s = 1, \ldots, n$ and $\ell = 1, \ldots, m$.

A3 (Smoothness & Kernel) The pdf $f_{YUZ}(y, u, z)$ and its marginals are continuously differentiable. All second-order derivatives of the function μ are continuous. The (auto-)covariance functions $\gamma_l((u_1, z_1), (u_2, z_2)) = \mathbb{E}(X^c_i(u_1, z_1)X^c_{i+l}(u_2, z_2))$, $l \geq 0$, are continuously differentiable for all points within their supports. The multiplicative kernel functions K_μ and K_γ are products of second-order kernel functions κ.

A4 (Moments & Dependency) X_i, U_{ij}, and Z_i are strictly stationary, ergodic, and weakly dependent time series with auto-covariances that converge uniformly to zero at a geometrical rate. It is assumed that $\mathbb{E}(X_i(u, z)^4) < \infty$, $\mathbb{E}(\epsilon_{ij}) = 0$, $\mathbb{E}(\epsilon^2_{ij}) = \sigma^2_\epsilon < \infty$ for all (u, z), i, and j.

A5 (Bandwidths) $h_{\mu,U}, h_{\mu,Z} \to 0$ and $(nm)h_{\mu,U}h_{\mu,Z} \to \infty$ as $nm \to \infty$. $h_{\mu,U}, h_{\mu,Z} \to 0$ and $(nM)h^2_{\mu,U}h_{\mu,Z} \to \infty$ as $nM \to \infty$.

Remark. Assumption A1 is a simplified version of the asymptotic setup of Zhang and Wang (2016). The case $\theta = 0$ implies that m is bounded which corresponds to a simplified version of the finite-m asymptotic considered by Jiang and Wang (2010). For $0 < \theta < \infty$ we can consider all further scenarios from sparse to dense functional data. In line with our real data application, we consider a deterministic m as also done, for instance, by Hall, Müller and Wang (2006). However, our results are generalizable to a random m using some minor modifications.

APPENDIX A: PROOFS

The following Lemma A.1 builds the basis of our theoretical results.

LEMMA A.1 (Bias and Variance of $\hat{\mu}$). *Let (u,z) be an interior point of $[0,1]^2$. Under Assumptions A1-A5 the conditional asymptotic bias and variance of the LLK estimator $\hat{\mu}$ in Eq. (2.2) are then given by*

(i) $\text{Bias}\{\hat{\mu}(u,z;h_{\mu,U},h_{\mu,Z})|\mathbf{U},\mathbf{Z}\} = B_\mu(u,z) + o_p(h^2_{\mu,U} + h^2_{\mu,Z})$ *with* $B_\mu(u,z) = \frac{1}{2}\,\nu_2(K_\mu)\,\left(h^2_{\mu,U}\,\mu^{(2,0)}(u,z) + h^2_{\mu,Z}\,\mu^{(0,2)}(u,z)\right)$, *where* $\mu^{(k,l)}(u,z) = (\partial^{k+l}/(\partial u^k \partial z^l))\mu(u,z)$.

(ii) $\mathrm{V}\{\hat{\mu}(u,z;h_{\mu,U},h_{\mu,Z})|\mathbf{U},\mathbf{Z}\} = \left(V^{\mathrm{I}}_\mu(u,z) + V^{\mathrm{II}}_\mu(u,z)\right)(1+o_p(1))$ *with* $V^{\mathrm{I}}_\mu(u,z) = (nm)^{-1}\left[h^{-1}_{\mu,U}h^{-1}_{\mu,Z}\,R(K_\mu)\dfrac{\gamma(u,u,z)+\sigma^2_\epsilon}{f_{UZ}(u,z)}\right]$ *and* $V^{\mathrm{II}}_\mu(u,z) = n^{-1}\left[\left(\dfrac{m-1}{m}\right)h^{-1}_{\mu,Z}\,R(\kappa)\dfrac{\gamma(u,u,z)}{f_Z(z)}\right]$.

Proof of Lemma A.1. Our proof of Lemma A.1 generally follows that of Ruppert and Wand (1994), and differs only from the latter reference as we consider additionally a conditioning variable Z_i, a function-valued error term, and a time series context.

Proof of Lemma A.1, part *(i)*: For simplicity, consider a second-order kernel function κ with compact support such as the Epanechnikov kernel; this is, of course, without loss of generality. Let (u,z) be a interior point of $[0,1]^2$ and define $\mathbf{H}_\mu = \mathrm{diag}(h^2_{\mu,U},h^2_{\mu,Z})$, $\mathbf{U} = (U_{11},\dots,U_{nm})^\top$, and $\mathbf{Z} = (Z_1,\dots,Z_n)^\top$. Using a Taylor-expansion of μ around (u,z), the conditional bias of the estimator $\hat{\mu}(u,z;\mathbf{H})$ can be written as

$$\begin{aligned}\text{(A.1)}\qquad & \mathbb{E}(\hat{\mu}(u,z;\mathbf{H}_\mu) - \mu(u,z)|\mathbf{U},\mathbf{Z}) = \\ & = \frac{1}{2}e_1^\top\left((nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]\right)^{-1}\times \\ & \times (nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}\left(\boldsymbol{\mathcal{Q}}_\mu(u,z) + \mathbf{R}_\mu(u,z)\right),\end{aligned}$$

where $\boldsymbol{\mathcal{Q}}_\mu(u,z)$ is a $nm\times 1$ vector with typical elements

$$(U_{ij}-u, Z_i-z)\boldsymbol{\mathcal{H}}_\mu(u,z)(U_{ij}-u,Z_i-z)^\top \in \mathbb{R}$$

with $\boldsymbol{\mathcal{H}}_\mu(u,z)$ being the Hessian matrix of the regression function $\mu(u,z)$. The $nm \times 1$ vector $\mathbf{R}_\mu(u,z)$ holds the remainder terms as in Ruppert and Wand (1994).

Next we derive asymptotic approximations for the 3×3 matrix $\left((nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]\right)^{-1}$ and the 3×1 matrix $(nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}\boldsymbol{\mathcal{Q}}_\mu(u,z)$ of the right hand side of Eq. (A.1). Using standard arguments from nonparametric statistics it is easy to derive that $(nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z] =$

$$\begin{pmatrix} f_{UZ}(u,z) + o_p(1) & \nu_2(K_\mu)\mathbf{D}_{f_{UZ}}(u,z)^\top \mathbf{H}_\mu + o_p(\mathbf{1}^\top \mathbf{H}_\mu) \\ \nu_2(K_\mu)\mathbf{H}_\mu \mathbf{D}_{f_{UZ}}(u,z) + o_p(\mathbf{H}_\mu \mathbf{1}) & \nu_2(K_\mu)\mathbf{H}_\mu f_{UZ}(u,z) + o_p(\mathbf{H}_\mu) \end{pmatrix},$$

where $\mathbf{1} = (1,1)^\top$ and $\mathbf{D}_{f_{UZ}}(u,z)$ is the vector of first order partial derivatives (i.e., the gradient) of the pdf f_{UZ} at (u,z). Inversion of the above block matrix yields

$$\left((nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]\right)^{-1} = \tag{A.2}$$

$$\begin{pmatrix} (f_{UZ}(u,z))^{-1} + o_p(1) & -\mathbf{D}_{f_{UZ}}(u,z)^\top (f_{UZ}(u,z))^{-2} + o_p(\mathbf{1}^\top) \\ -\mathbf{D}_{f_{UZ}}(u,z)\,(f_{UZ}(u,z))^{-2} + o_p(\mathbf{1}) & (\nu_2(K_\mu)\mathbf{H}_\mu f_{UZ}(u,z))^{-1} + o_p(\mathbf{H}_\mu) \end{pmatrix}.$$

The 3×1 matrix $(nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}\boldsymbol{\mathcal{Q}}_\mu(u,z)$ can be partitioned as following:

$$(nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}\boldsymbol{\mathcal{Q}}_\mu(u,z) \quad = \quad \begin{pmatrix} \texttt{upper element} \\ \texttt{lower bloc} \end{pmatrix},$$

where the 1×1 dimensional `upper element` can be approximated by

$$\begin{aligned} &(nm)^{-1}\sum_{ij} K_{\mu,h}(U_{ij}-u, Z_{\mathrm{I}}-z)(U_{ij}-u, Z_{\mathrm{I}}-z)\boldsymbol{\mathcal{H}}_\mu(u,z)(U_{ij}-u, Z_{\mathrm{I}}-z)^\top \\ &= (\nu_2(\kappa))^2\, tr\left\{\mathbf{H}_\mu \boldsymbol{\mathcal{H}}_\mu(u,z)\right\} f_{UZ}(u,z) + o_p(tr(\mathbf{H}_\mu)) \end{aligned} \tag{A.3}$$

and the 2×1 dimensional `lower bloc` is equal to

$$\begin{aligned} &(nm)^{-1}\sum_{ij}\left\{K_{\mu,h}(U_{ij}-u, Z_{\mathrm{I}}-z)(U_{ij}-u, Z_{\mathrm{I}}-z)\boldsymbol{\mathcal{H}}_\mu(u,z)(U_{ij}-u, Z_i-z)^\top\right\} \times \\ &\times (U_{ij}-u, Z_{\mathrm{I}}-z)^\top = O_p(\mathbf{H}_\mu^{3/2}\mathbf{1}). \end{aligned} \tag{A.4}$$

Plugging the approximations of Eqs. (A.2)-(A.4) into the first summand of the conditional bias expression in Eq. (A.1) leads to the following expression

$$\begin{aligned}&\frac{1}{2}e_1^\top\left((nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]\right)^{-1}\times\\&\times(nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}\boldsymbol{\mathcal{Q}}_\mu(u,z)=\\&=\frac{1}{2}\left(\nu_2(\kappa)\right)^2 tr\left\{\mathbf{H}_\mu\boldsymbol{\mathcal{H}}_\mu(u,z)\right\}+o_p(tr(\mathbf{H}_\mu)).\end{aligned}$$

Furthermore, it is easily seen that the second summand of the conditional bias expression in Eq. (A.1), which holds the remainder term, is given by

$$\begin{aligned}&\frac{1}{2}e_1^\top\left((nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]\right)^{-1}\times\\&\times(nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}\mathbf{R}_\mu(u,z)=o_p(tr(\mathbf{H}_\mu)).\end{aligned}$$

Summation of the two latter expressions yields the asymptotic approximation of the conditional bias

$$\mathbb{E}(\hat{\mu}(u,z;\mathbf{H}_\mu)-\mu(u,z)|\mathbf{U},\mathbf{Z})=\frac{1}{2}\left(\nu_2(\kappa)\right)^2 tr\left\{\mathbf{H}_\mu\boldsymbol{\mathcal{H}}_\mu(u,z)\right\}+o_p(tr(\mathbf{H}_\mu)).$$

Proof of Lemma A.1, part *(ii)*: In the following we derive the conditional variance of the local linear estimator $\mathrm{V}(\hat{\mu}(u,z;\mathbf{H}_\mu)|\mathbf{U},\mathbf{Z})=$

$$\begin{aligned}=&e_1^\top\left([\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]\right)^{-1}\times\\&\times[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}\,\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\,\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]\times\\&\times([\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z])^{-1}e_1\\=&e_1^\top((nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z])^{-1}\times\\&\times((nm)^{-2}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}\,\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\,\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z])\times\\&\times((nm)^{-1}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z])^{-1}e_1,\end{aligned}\tag{A.5}$$

where $\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})$ is a $nm\times nm$ matrix with typical elements

$$\begin{aligned}\mathrm{Cov}(Y_{ij},Y_{\ell k}|U_{ij},U_{\ell k},Z_i,Z_\ell)=&\gamma_{|i-\ell|}((U_{ij},Z_i),(U_{\ell k},Z_\ell))+\\&+\sigma_\epsilon^2\mathbb{1}(i=\ell\text{ and }j=k);\end{aligned}$$

with $\mathbb{1}(.)$ being the indicator function.

We begin with analyzing the 3×3 matrix

$$(nm)^{-2}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top\mathbf{W}_{\mu,uz}\,\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\,\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]$$

using the following three Lemmas A.2-A.4.

Lemma A.2. *The upper-left scalar (block) of the matrix* $(nm)^{-2}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]$ *is given by*

$$
\begin{aligned}
& (nm)^{-2}\mathbf{1}^\top \mathbf{W}_{\mu,uz}\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\mathbf{W}_{\mu,uz}\mathbf{1} \\
= \quad & (nm)^{-1} f_{UZ}(u,z)|\mathbf{H}_\mu|^{-1/2}R(K_\mu)\left(\gamma(u,u,z)+\sigma_\epsilon^2\right)(1+O_p(tr(\mathbf{H}_\mu^{1/2}))) \\
+ \quad & n^{-1}(f_{UZ}(u,z))^2\left[\left(\frac{m-1}{m}\right)h_{\mu,Z}^{-1}R(\kappa)\frac{\gamma(u,u,z)}{f_Z(z)}+c(u,z)\right](1+O_p(tr(H^{1/2}))) \\
= \quad & O_p((nm)^{-1}|\mathbf{H}_\mu|^{-1/2})+O_p(n^{-1}h_{\mu,Z}^{-1}),
\end{aligned}
$$

where $c(u,z)=2\sum_{l=1}^{n-1}\gamma_l((u,z),(u,z))$. *Under Assumption A4 there exists a constant* C, $0<C<\infty$, *such that* $0\le |c(u,z)|\le C$.

Lemma A.3. *The* 1×2 *dimensional upper-right block of the matrix* $(nm)^{-2}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]$ *is given by*

$$
\begin{aligned}
& (nm)^{-2}\mathbf{1}^\top \mathbf{W}_{\mu,uz}\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\mathbf{W}_{\mu,uz}\begin{pmatrix}(U_{11}-u,Z_1-z)\\ \vdots \\ (U_{nm}-u,Z_n-z)\end{pmatrix} \\
= \quad & (nm)^{-1} f_{UZ}(u,z)|\mathbf{H}_\mu|^{-1/2}(\mathbf{1}^\top\mathbf{H}_\mu^{1/2})R(K_\mu)\left(\gamma(u,u,z)+\sigma_\epsilon^2\right)(1+O_p(tr(\mathbf{H}_\mu^{1/2}))) \\
+ \quad & n^{-1}(f_{UZ}(u,z))^2(\mathbf{1}^\top\mathbf{H}_\mu^{1/2})\left[\left(\frac{m-1}{m}\right)h_{\mu,Z}^{-1}R(\kappa)\frac{\gamma(u,u,z)}{f_Z(z)}+c_r\right](1+O_p(tr(\mathbf{H}_\mu^{1/2}))) \\
= \quad & O_p((nm)^{-1}|\mathbf{H}_\mu|^{-1/2}(\mathbf{1}^\top\mathbf{H}_\mu^{1/2}))+O_p(n^{-1}(\mathbf{1}^\top\mathbf{H}_\mu^{1/2})h_{\mu,Z}^{-1}),
\end{aligned}
$$

where $c(u,z)=2\sum_{l=1}^{n-1}\gamma_l((u,z),(u,z))$. *Under Assumption A4 there exists a constant* C, $0<C<\infty$, *such that* $0\le |c(u,z)|\le C$.

Remark. *The* 2×1 *dimensional lower-left block of the matrix* $(nm)^{-2}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\mathbf{W}_{\mu,uz}[\mathbf{1},\mathbf{U}_u,\mathbf{Z}_z]$ *is simply the transposed version of the result in Lemma A.3.*

LEMMA A.4. *The 2×2 lower-right block of the matrix* $(nm)^{-2}[\mathbf{1}, \mathbf{U}_u, \mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\mathbf{W}_{\mu,uz}[\mathbf{1}, \mathbf{U}_u, \mathbf{Z}_z]$ *is given by*

$$
\begin{aligned}
& (nm)^{-2}\left(((U_{11}-u),(Z_1-z))^\top, \ldots, ((U_{nm}-u),(Z_n-z)^\top)\right) \times \\
& \times \mathbf{W}_{\mu,uz}\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\mathbf{W}_{\mu,uz}\begin{pmatrix}(U_{11}-u, Z_1-z)\\ \vdots \\ (U_{nm}-u, Z_n-z)\end{pmatrix} \\
= \; & (nm)^{-1} f_{UZ}(u,z)|\mathbf{H}_\mu|^{-1/2}\mathbf{H}_\mu R(K_\mu)\left(\gamma(u,u,z)+\sigma_\epsilon^2\right)(1+O_p(tr(\mathbf{H}_\mu^{1/2}))) \\
+ \; & n^{-1}(f_{UZ}(u,z))^2\mathbf{H}_\mu\left[\left(\frac{m-1}{m}\right)h_{\mu,Z}^{-1}R(\kappa)\frac{\gamma(u,u,z)}{f_Z(z)}+c_r\right](1+O_p(tr(\mathbf{H}_\mu^{1/2}))) \\
= \; & O_p((nm)^{-1}|\mathbf{H}_\mu|^{-1/2}\mathbf{H}_\mu)+O_p(n^{-1}\mathbf{H}_\mu h_{\mu,Z}^{-1}),
\end{aligned}
$$

where $c(u,z) = 2\sum_{l=1}^{n-1}\gamma_l((u,z),(u,z))$*. Under Assumption A4 there exists a constant* C*,* $0 < C < \infty$*, such that* $0 \leq |c(u,z)| \leq C$*.*

Using the approximations for the bloc-elements of the matrix $(nm)^{-2}[\mathbf{1}, \mathbf{U}_u, \mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\mathbf{W}_{\mu,uz}[\mathbf{1}, \mathbf{U}_u, \mathbf{Z}_z]$, given by the Lemmas A.2-A.4, and the approximation for the matrix $\left((nm)^{-1}[\mathbf{1}, \mathbf{U}_u, \mathbf{Z}_z]^\top \mathbf{W}_{\mu,uz}[\mathbf{1}, \mathbf{U}_u, \mathbf{Z}_z]\right)^{-1}$, given in (A.2), we can approximate the conditional variance of the bivariate local linear estimator, given in (A.5). Some straightforward matrix algebra leads to $\mathrm{V}(\hat{\mu}(u,z;\mathbf{H}_\mu)|\mathbf{U},\mathbf{Z}) =$

$$
\begin{aligned}
& (nm)^{-1}|\mathbf{H}_\mu|^{-1/2}\left\{\frac{R(K_\mu)\left(\gamma(u,u,z)+\sigma_\epsilon^2\right)}{f_{UZ}(u,z)}\right\}(1+o_p(1)) \\
& + n^{-1}\left[\left(\frac{m-1}{m}\right)h_{\mu,Z}^{-1}R(\kappa)\frac{\gamma(u,u,z)}{f_Z(z)}+c_r\right](1+o_p(1)),
\end{aligned}
$$

which is asymptotically equivalent to our variance statement of Lemma A.1 part *(ii)*.

Proof of Lemma A.2. (The proofs of Lemmas A.3 and A.4 can be done correspondingly.) To show Lemma A.2 it will be convenient to split the sum such that $(nm)^{-2}\mathbf{1}^\top \mathbf{W}_{\mu,uz}\mathrm{Cov}(\mathbf{Y}|\mathbf{U},\mathbf{Z})\mathbf{W}_{\mu,uz}\mathbf{1} = s_1 + s_2 + s_3$. Using standard procedures from kernel density estimation leads to

(A.6)

$$
\begin{aligned}
s_1 &= (nm)^{-2}\sum_{ij}(K_{\mu,h}(U_{ij}-u, Z_{\mathrm{I}}-z))^2\,\mathrm{V}(Y_{ij}|\mathbf{U},\mathbf{Z}) \\
&= (nm)^{-1}|\mathbf{H}_\mu|^{-1/2}f_{UZ}(u,z)R(K_\mu)\left(\gamma(u,u,z)+\sigma_\epsilon^2\right)+O((nm)^{-1}|\mathbf{H}_\mu|^{-1/2}\,tr(\mathbf{H}_\mu^{1/2}))
\end{aligned}
$$

(A.7)

$$
\begin{aligned}
s_2 &= (nm)^{-2}\sum_{jk}\sum_{\substack{i\ell\\ i\neq\ell}}K_{\mu,h}(U_{ij}-u, Z_{\mathrm{I}}-z)\,\mathrm{Cov}(Y_{ij},Y_{\ell k}|\mathbf{U},\mathbf{Z})\,K_{\mu,h}(U_{\ell k}-x, Z_\ell-z) \\
&= n^{-1}(f_{UZ}(u,z))^2c(u,z)+O_p(n^{-1}tr(\mathbf{H}_\mu^{1/2}))
\end{aligned}
$$

(A.8)

$$s_3 = (nm)^{-2}\sum_{\substack{ij\\ i\neq j}}\sum_{t} h_{\mu,U}^{-1}\kappa(h_{\mu,U}^{-1}(U_{ij}-u))(h_{\mu,Z}^{-1}\kappa(h_{\mu,Z}^{-1}(Z_{\mathrm{I}}-z)))^2\,\mathrm{Cov}(Y_{ij},Y_{jt}|\mathbf{U},\mathbf{Z})\times$$
$$\times\, h_{\mu,U}^{-1}\kappa(h_{\mu,U}^{-1}(U_{ik}-x))$$
$$= n^{-1}(f_{UZ}(u,z))^2\left[\left(\frac{m-1}{m}\right)h_{\mu,Z}^{-1}R(\kappa)\frac{\gamma(u,u,z)}{f_Z(z)}\right] + O_p(n^{-1}tr(\mathbf{H}_\mu^{1/2})),$$

where $c(u,z) = 2\sum_{l=1}^{n-1}\gamma_l((u,z),(u,z))$. Summing up (A.6)-(A.7) leads to the result in Lemma A.2. Lemmas A.3 and A.4 differ from Lemma A.2 only with respect to the additional factors $\mathbf{1}^\top\mathbf{H}_\mu^{1/2}$ and $\mathbf{H}_\mu$. These come in due to the usual substitution step for the additional data parts $(U_{ij}-u, Z_i-z)$.

Proofs of Theorem 2.1 and Corollary 2.1. Theorem 2.1 and Corollary 2.1 follow directly from Lemma A.1 and from applying a central limit theorem for strictly stationary ergodic times series such as Theorem 9.5.5 in Karlin and Taylor (1975).

APPENDIX B: DATA SOURCES

Hourly spot prices of the German electricity market are provided by the European Energy Power Exchange (EPEX) (`www.epexspot.com`), hourly values of Germany's gross electricity demand and electricity exchanges with other countries are provided by the European Network of Transmission System Operators for Electricity (`www.entsoe.eu`). German wind and solar power infeed data are provided by the transparency platform of the European Energy Exchange (`www.eex-transparency.com`). German air temperature data are available from the German Weather Service (`www.dwd.de`). The daily prices for natural gas are provided via the trading platform PEGAS, which is part of the European Energy Exchange (EEX) Group operated by Powernext (`www.powernext.com`). Daily prices for European CO_2 Emission Allowances (EUA) and for the Amsterdam-Rotterdam-Antwerp (ARA) coal futures are provided via the websites of the EEX (`www.eex.com`)

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